Quantification

If people or animals stare at a landscape, they can understand a lot simply from what their senses are registering, but philosophers need to talk about it, and hence take a keen interest in how we refer to things. Quantification is one of the tools that is seen as crucial to talking clearly about the world. In standard logic an argument is about a 'domain' of objects, and quantifiers say which parts of the domain are being referred to.

Quantifiers are also part of ordinary language, since we can ask 'what are we talking about?', which typically suggests a topic (or domain), and an aspect of that topic or domain (such as all, or part, or one member, or none of it). Ancient logic focused on sentences which combined 'terms' (such as 'that man' and 'brave') which were combined by a 'copula' (such as 'is'). The quantification was part of the copula, which we can express as 'all-are', 'none-are', 'some-are' and 'some-are-not'. Modern logic began by extracting the quantification from the copula, and presenting it as an operator acting on a sentence, using variables to refer to ingredients of the sentence. The symbols ∀ (the **universal** quantifier 'for all') and ∃ (the **existential** quantifier 'for some') were introduced, so that ∀xFx says 'all x's are F', and ∃xFx says 'some x is F' (which can thus express generalisations). Such 'atomic' sentences can then be combined (using 'and', 'or' etc) for form more complex statements than ancient logic could handle. A vital tool in such complexity was to indicate the '**scope**' of each quantifier (what exactly it referred to), so brackets (borrowed from mathematics) became an important part of the symbolism. We talk of 'quantifying **over**' the predicate of a sentence, and a variable is said to be either 'free', or '**bound**' by a quantifier; once variables are bound, truth-values can be assigned.

Since 'there does not exist an x which is not F' means 'all x's are F', and 'not all x's are not F' means 'some x is F', the quantifiers are interdefinable (with negation), so a language just needs the universal quantifier. This refers to 'all', but the semantics of logic usually employs set theory, and the universal set (containing everything) is a logic impossibility, so 'all' just refers to the domain (though multiple domains of different 'type' are possible). How we can then reason about 'everything' is a problem which bothers logicians. The obvious question about the existential quantifier is whether it actually implies true existence (as part of our 'ontological commitment'). Some logicians propose to give it a less loaded name, or introduce 'free logic' in which quantifiers do not imply existence at all, or to just focus on the role of quantifiers in reasoning.

If we are quantifying over a domain of objects, then to refer to all of them (universal quantification) might be understood as 'the first, *and* the second, *and* the third, etc…', thus eliminating the quantifier in favour of a huge **conjunction**. Similarly, the existential quantifier says 'the first, *or* the second, *or* the third…', suggesting a huge **disjunction**. In this way, quantifiers are not indispensable in standard logic, but (crucially) this is only for a finite domain of objects, and will thus not deal with uncountable numbers. If a domain only had three objects, we probably wouldn't bother with quantifiers.

One response to the ontological problems of quantifiers is to not treat them in their usual **objectual** way (where variables refer to objects), but to treat them as **substitutional**, where the variables refer to linguistic entities, such as names and singular terms. The standard problems with the interpretation of quantifiers then seem to be avoided, but other problems arise. In particular, the logic was designed for mathematics, and if we are quantifying over the uncountable real numbers there simply can't be enough names or terms available to do the job. The objectual approach can talk of 'all' objects, but the substitutional approach can't cope with unnamed objects. On the other hand, substitutional quantification shifts attention away from 'satisfaction' by objects (which may be dubiously abstract), and focuses instead on the truth of sentences, leaving the ontology to be settled on another day.

An immediate consequence of a universal quantification is that if something is true of all members of the domain, it must be true of one member, so ∀xF(x) implies F(t) ('they're all F, so something is F'), where 't' is a term denoting an object. This '**universal instantiation**' is a basic rule of quantification. The existential quantifier has the parallel rule of '**existential generalisation**', since 'Fido is angry' implies 'something is angry'.

If we quantify over a domain of objects, this is 'first-order' logic. If we stop there, the ontology is tidy, and it gives a reasonable logic for the sciences (by treating properties as sets of objects), but mathematicians need logic to be '**second-order**' to fully capture the reasoning of their subject. A second-order quantifier typically applies variables to properties, relations and functions. These are either understood as belonging to a single second-order domain, or are separated into two groups, each with its own variables. The quantifiers themselves do not change their meaning in second-order logic, but they quantify over different things.

The two quantifiers are considered sufficient for classical logic, but further quantifiers or alternative quantifiers are sometimes thought to be helpful. If we want to indicate the uniqueness of an object, we can write '∃!xFx', meaning 'there is just one thing which is F'. Other suggested quantifiers include 'uncountable', 'two-argument', 'intentional', and 'branching' quantifiers.

'We walked to work' implies 'I walked to work', but 'we surrounded the car' doesn't imply 'I surrounded the car'. The predicate 'surround' is said to be 'non-distributive' (since it doesn't apply to the individuals), and 'multigrade' (since it doesn't need a fixed number of people), and it is suggested that **plural quantification** is needed to handle these, and such complex statements as 'some critics only admire one another'. To say 'some cars are unreliable' seems to fall between existential and universal quantification. Thus we retain the quantifiers, but write ∃xxFxx to mean 'some x's are F', and introduce a symbol to express 'y is one of the x's'. This seems to offer alternative approaches to set theory and to second-order logic, while retaining an ontology that is close to ordinary talk. Critics ask whether there might be a hidden ontology (of collections) in the seemingly innocent reference to some objects. Plural quantification offers the possibility of reasoning about a ship as 'some atoms', without having to pin down which atoms they are, or how many of them are involved.